Questions or Review Issues for Week 2

For a block of signal samples [x(0), x(1), x(2), x(3), x(4), x(5),x(6), x(7)]:

1. Write the 8-point DFT of the block;
2. Work out the system diagram for the DFT when using the radix-2 decimation-in-time FFT algorithm;
3. Work out the system diagram for the DFT when using the radix-2 decimation-in-frequency FFT algorithm;
4. Work out the system diagram for the DFT when using the radix-4 decimation-in-frequency FFT algorithm;
5. Compare the number of multiplications required for computing the DFT using the three approaches above.

Solution:

1. $X(k) = \sum_{n=0}^{7} x(n) W_8^{nk}$

2. refer to lecturenotes

3. refer to Lecturenotes

4. $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk} = \sum_{n=0}^{\frac{N}{4}-1} \left[\sum_{l=0}^{3} x(n + l\frac{N}{4}) W_N^{\frac{N}{4}k} \right] W_N^{nk} = \sum_{n=0}^{\frac{N}{4}-1} \left[\sum_{l=0}^{3} x(n + l\frac{N}{4}) (-j)^{lk} \right] W_N^{nk}$

Splitting X(k) into 4 sub-blocks X(4k+m) (for m=0, 1,2, 3; and k=0, 1, …, (N/4 –1)), we have $X(4k+m) = \sum_{n=0}^{\frac{N}{4}-1} \left[\sum_{l=0}^{3} (-j)^{lm} x(n + l\frac{N}{4}) W_N^{nm} \right] W_{N/4}^{nk}$

For N=8, the above equation becomes:

$X(4k+m) = \sum_{n=0}^{1} \left[\sum_{l=0}^{3} (-j)^{lm} x(n+2l) W_8^{nm} \right] W_2^{nk}$ (for m=0,1,2,3; and k=0,1)

Let $y(m,n) = \sum_{l=0}^{3} (-j)^{lm} x(n+2l) W_8^{nm}$, then $X(4k+m) = \sum_{n=0}^{1} y(m,n) W_2^{nk}$,

Hence the 8-point DFT can be considered as:

a. X(4k) (X(0)&X(4)) are $X(4k) = \sum_{n=0}^{1} y(0,n) W_2^{nk}$ the 2-point DFT of y(0,n);

b. X(4k+1) (X(1)&X(5)) are $X(4k+1) = \sum_{n=0}^{1} y(1,n) W_2^{nk}$ the 2-point DFT of y(1,n);

c. X(4k+2) (X(2)&X(6)) are $X(4k+2)=\sum_{n=0}^{1} y(2,n)W_2^{nk}$ the 2-point DFT of y(2,n);

d. X(4k+3) (X(3)&X(7)) are $X(4k+2)=\sum_{n=0}^{1} y(2,n)W_2^{nk}$ the 2-point DFT of y(3,n);

Hence the 8-point DFT can be implemented as two stages:

Stage 1: Construct y(m,n) based on $y(m,n)=\sum_{l=0}^{3}(-j)^{lm}x(n+2l)W_8^{nm}$;

Stage 2: Compute the 2-point DFTs above.

The system diagram can be obtained accordingly.

6. Computation comparison:
   a. Radix 4 approach: for constructing y(m,n), for each pair of (m,n) one multiplication is required. sine (n=0,1; and m=0,1,2,3) and hence 3 multiplications are required (excluding the cases n=0 or m=0) for constructing y(m,n). Then the second stage consists of 4 2-point DFT, no multiplication is required for 2-point DFT. Hence in total 3 multiplications are required;
   b. Radix 2 algorithms contain three stages and 5 multiplications are required.